REVISED RECOMMENDED METHODS FOR ANALYZING CRATER STATISTICS.

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Introduction: The modern field of impact crater studies has seen only one main attempt to standardize how crater population data are displayed and presented [1]. Since that work, not only has the field progressed, but so has computer power, display capabilities, and relevant statistical methods. The field has generally refrained from using many of the statistical tools available to them, and instead, as a starting point, rely on now-traditional treatment that made many simplifying assumptions. Therefore, different researchers often deal with crater data differently, showing what appear to be standardized graphs but in fact contain differences that can change how the data are interpreted.

This was one of the driving motivations behind the 2015 "Workshop on Issues in Crater Studies and the Dating of Planetary Surfaces." During the workshop, the issue of standardization was raised, as was proper treatment of impact crater statistics. Since that time, several of us have worked together to investigate the problem and develop techniques and recommendations. We hope to spark discussion that will lead to a new, inclusive document in the near future that can be referenced by both experienced and novice crater analysts.

Philosophical Approach: Traditionally, craters are treated as discrete objects that either exist or do not, and they are measured in an objective manner to yield their properties. As an alternative, we propose craters be thought of as a sample from a probability distribution: Impact craters are drawn from an unknown probability distribution, and they are affected by deterministic and stochastic processes; moreover, "crater counts" (a term that also carries baggage) are the analyst's sampling of a population which is already a sample of the underlying processes. From the "counts," we want to work backwards to understand the probability distribution from which the craters were formed and processes that have have affected the distribution (understanding there are sampling issues in our measurement, as well).

Sources of Uncertainty: Sources of uncertainty are often known within the crater community but are rarely discussed. While researcher variability was known at least as early as 1970 [2], they are not factored into error bars nor standard crater analyses. Similarly, while crater diameter measurement has uncertainty (from technique, reproducibility by one analyst, or replicability by others), this is also not factored in.

Display Change: Thinking of impact crater observations from a probability distribution, with numerous sources of bias and uncertainty, which themselves sample another probability distribution, suggest the data be displayed as a "Probability Density Function" (PDF) which is built by the "Kernel Density Estimator" (KDE) technique [3,4]. This creates the same distribution as the traditional "differential" plot from [1].

The KDE models each impact crater i as a normalized probability distribution (*e.g.*, a Gaussian) with μ_i at the observed crater diameter D_i and σ_i as an uncertainty in that crater diameter. Research has shown that crater diameter replicability is ≈10% and *does* approximately follow a Gaussian distribution [2,5,6], so this can be used as σ ($\sigma_i = 0.1D_i$). To construct the KDE, one determines where to sample diameters (d_s) to create a <u>visually</u> continuous KDE (since KDEs are continuous, unbound distributions) and then calculates the sum of all craters' probability distributions at each d_s.

Once the PDF is constructed, it can be summed from large to small diameters to create the traditional cumulative plot (CSFD), or divided by D^{-3} to create the traditional R plot (Fig. 1). Except for very sparse crater datasets (*e.g.*, at large diameters relative to the rest of the data), the PDF will reproduce traditionally binned data almost exactly. Differences in sparse data are due to binning acting as a more significant smoother than the KDE and individual techniques' variations in where to place the diameter bin. The former can be demonstrated, in part, visually by adding what is known as a "rug plot" to the horizontal axis which shows a small symbol for every D_i.

Confidence Envelope: Traditionally, crater analysts use 1-sigma error bars based on Poisson counting uncertainty ($\sigma = \text{sqrt}(N)$, where N is the number of craters sampled). This σ is based solely upon counting statistics; therefore, it does not include any other sources described in this abstract. Also, depending on the purpose of a CSFD, uncertainties are incorrectly represented as sqrt(N) for they do not include the uncertainty of counts in individual bins.

Statistical literature provides a Monte Carlo technique, known as the bootstrap, for calculating confidence intervals (CIs) for some functions of our crater data [7]. The bootstrap has the benefit of working in situations where one is not able or not willing to assume an underlying structure (*e.g.*, Gaussian) for the probability distribution. The basic idea behind the bootstrap is to generate M bootstrap datasets where M is large (*e.g.*, ≥1000; the recommendation has increased over the years as computing power has increased). Mechanically, for each bootstrap dataset m_i, N (the size of the original crater dataset) crater diameters are drawn with replacement from the PDF at random; because the method is "with replacement," one may sample the same crater more than once. A PDF is created for each m_i. For each d_s, the new PDF's value is stored. After M runs, at each d_s, sort the M values of the Monte Carlo PDFs. From the PDF of the observed population, the value corresponding to the desired confidence interval above and below it would be recorded for each d_s. *E.g.*, if CI = 95%, M = 100, and at a given d_s the observed PDF is at the 60th position in the sorted list, then the CI for that d_s corresponds to the 3rd value and 98th value (60×(1–0.95) and 60+(100–60)×0.95).

This method removes assumptions about the distribution (*e.g.*, Poisson), assumptions about symmetry of the confidence bands, and emphasizes that the crater population has a confidence band rather than error bars on specific bins. We are exploring varying N for each m_i to simulate repeatability variations [*e.g.*, 5].

Fitting a Power-Law: By default, most crater analysts will use whatever their analysis software of choice has to fit a power law to their (binned) data. This is most often a form of linear or non-linear least-squares (LS) (note— linear LS does not mean the fit takes the form of a line). If one were to use this with the PDF, we recommend re-sampling the PDF at each original crater diameter as the x value and taking the PDF±CI at that point as the $y \pm \sigma_y$ value.

However, it has long been known in the statistics literature and the crater literature [8] that LS produces biased results, especially in data that are non-uniform or non-Gaussian because a core assumption of LS is δy_i are independent and identically distributed. A significantly less biased method is the Maximum Likelihood Estimator (MLE). MLE uses the original crater diameters and does not rely on any binning or smoothing to produce the fit parameter. There is a simple analytic form of the MLE for a Pareto distribution (power law truncated at a minimum diameter), and a version for a truncated Pareto distribution (bound at large and small diameters) exists but must be solved numerically. MLE has the added benefit that it does not rely on the data display or format, so error bars on the MLE are the same regardless of whether the data are represented as a differential, CSFD or other form.

We used both a frequentist and Bayesian likelihood estimate, as well as LS to the resampled PDF and traditional bins, to fit power laws to data randomly sampled from a power law distribution with differential slope –3 for N craters M times, where N = 5–905 and M = 2500–40,000 (depending on N; Bayesian prior set to –3 exponent with small level of Gaussian noise added). The results (Fig. 2) demonstrate the likelihood estimates produce the true slope with less bias and less variance than LS fitting. When using LS fitting, the fit to the PDF reproduced the true power law with less bias and variance than using the traditional bins. Finally, in cases of very small N where binning is practically meaningless, the MLE *always* returned a fit exponent, while the binned data often returned an inconclusive fit (exponent $\notin$ (–13,+7)).

Summary: We have presented several new suggestions for the treatment of crater population data that highlights some of our recent work. We think the methods described herein are a quantitative treatment of important characteristics of crater population data, factoring in many known but previously ignored uncertainties, and are a better statistical treatment of the field.

References: [1] Crater Analysis Techniques Working Group (1979) doi:10.1016/0019-1035(79)90009-5. [2] Greeley & Gault (1970) doi:10.1007/BF00561875. [3] Rosenblatt (1956) doi:10.1214/aoms/1177728190. [4] Parzen (1962) doi:10.1214/aoms/1177704472. [5] Robbins *et al.* (2014) doi:10.1016/j.icarus.2014.02.022. [6] Robbins *et al.* (in rev.) "Craters of the Pluto-Charon System." [7] Efron (1979) doi:10.1214/aos/1176344552. [8] Chapman & Haefner (1967) doi:10.1029/JZ072i002p00549.

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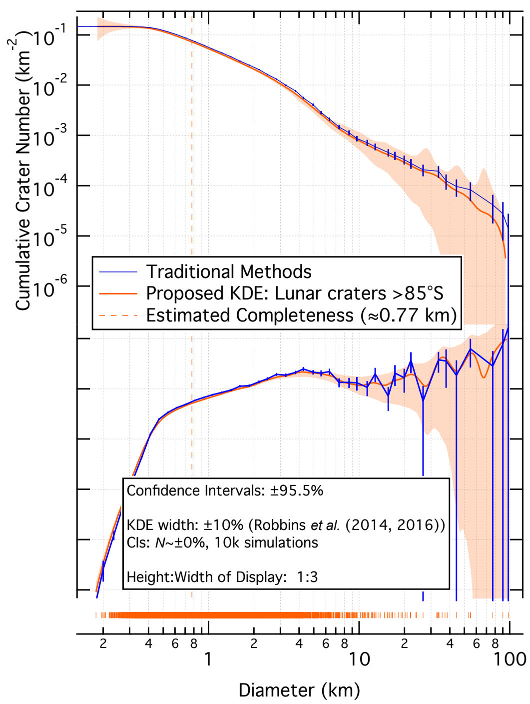

Figure 1. Example CSFD and R-plot with the traditional bins and new PDF. This illustrates several of our recommendations, both stated in this text (*e.g.*, rug plot) and not (*e.g.*, dotted vertical line estimating the completeness limit).

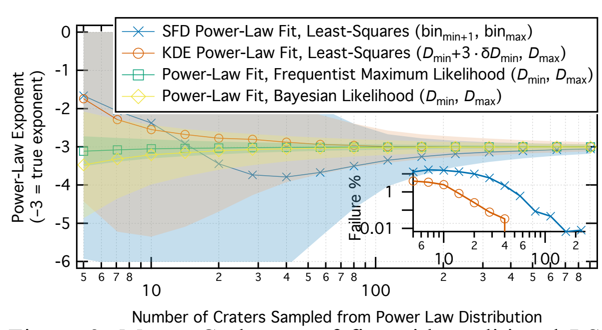

Figure 2: Monte Carlo test of fits with traditional LS and recommended LE; failure ≡ exponent $\notin$ (–13,+7).