**METEORITES ON EARTH AND MARS: THEORETICAL AND OBSERVATIONAL ANALYSIS WITH IMPACT RATE PROBABILITY.**

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**Introduction:** A framework for the theoretical and analytical understanding of the impact crater-size frequency distribution is developed and applied to observed data from Mars and Earth. The analytical model derived gives the crater population as a function of crater diameter, $D$, and age, $\tau$, taking into consideration the reduction in crater number as a function of time, caused by the elimination of craters due to effects such as erosion, obliteration by other impacts, and tectonic changes. When applied to Mars, using Barlow's impact crater catalog[1] (Figures (1)), we are able to determine an analytical curve ,in Figure (2) and Eqs.(1) to (4), describing the number of craters per bin size, $N(D)$, which perfectly reproduces and explains the presence of two well-defined slopes in the $\log[N]\ vs \log[D]$ plot. We see that the theoretical curve shown in Figure (2) differs significantly from the observed data for $D$ less than about $8km$. However, according to Barlow[1], her empirical data undercounts the actual crater population for $D$ less than $8km$ and therefore, we will restrict our analysis to $D \geq 8km$.

$$N = \bar{\Phi}\tau_{\text{mean}}\{1 - Exp[-\tau_f/\tau_{mean}]\}, \quad 1$$

$$\bar{\Phi}\tau_{\text{mean}} = \frac{1.43x10^5}{D^{1.8}}, \quad 2$$

$$\tau_f/\tau_{mean} = \frac{2.48x10^4}{D^{2.5}}, \quad 3$$

$$\bar{\Phi} = \left(\frac{1}{\tau_f}\right)\int_0^{\tau_f}\Phi d\tau = \left(\frac{1}{\tau_f}\right)\frac{3.55x10^9}{D^{4.3}}. \quad 4$$

Above, $\bar{\Phi}(D)$ is the time average( over the total time of crater formation $\tau_f$) of the rate of meteorite impacts per bin, $\Phi(D)$, capable of forming craters of diameter $D$. Also, $\tau_{mean}$ is the mean-life of craters of diameter $D$, since it can be shown [2] that $Exp[-\tau/\tau_{mean}]$ is the fraction of craters surviving today, that were formed at time $\tau$ ago. We see from Eq. (3) that craters with $D \approx 57km$ have $\tau_{mean} \approx \tau_f$, whereas $\tau_{mean} \gg \tau_f,\ if\ D \gg 57km$, and $\tau_{mean} \ll \tau_f,\ if\ D \ll 57km$. In the limit $D \gg 57km$, $\tau_f/\tau_{mean} \ll 1$, we obtain, from Eqs. (1) and (4), that:

$$N = \bar{\Phi}\tau_f = \frac{3.55x10^9}{D^{4.3}};\ \tau_f/\tau_{mean} \ll 1,\ D \gg 57km, \quad 5$$

which corresponds to a straight line of slope -4.3 in the $\log(N)$ vs. $\log(D)$ plot, that we see in the right-hand part of Figure (2), and is the form of Eq. (1) when we can ignore the destruction of craters. In other words, for these larger craters, their number is simply given by the expected relationship: $N = \bar{\Phi}\tau_f \equiv \int_0^{\tau_f}\Phi d\tau$, when craters are conserved and therefore, when the actual crater number is proportional to the age of the underlying surface $\tau_f$. On the other hand, for smaller craters where $\tau_f/\tau_{mean} \gg 1$ we will have, from Eqs. (1) and (2), that

$$N = \bar{\Phi}\tau_{mean} = \frac{1.43x10^5}{D^{1.8}},\ \tau_f/\tau_{mean} \gg 1, D \ll 57km, \quad 6$$

and hence in this limit, $N$ is proportional to the survival mean-life, $\tau_{mean}$, of craters of size $D$. This feature was called the 'crater retention age'by Hartmann [3], and on Mars is shown in craters with $D$ less than about $57km$, corresponding to the straight line segment on the left-hand side of Figure (2) with slope -1.8. Therefore, the above model tells us that the empirical curve is essentially constructed by the two straight lines in the $\log N(D)\ vs \log D$ plot given by Eqs.(5) and (6).The exponent 4.3 is pristine, while the exponent 1.8 is the result of a steady state equilibrium between elimination and creation of craters. The large exponent, 4.3, has interesting implications for the corresponding impactor size-frequency distribution, and we elaborate on this topic below.

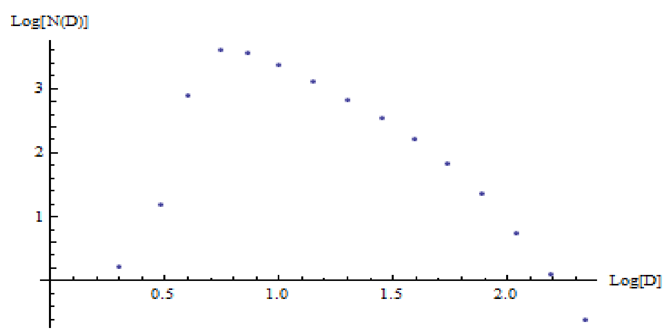


FIGURE (1): Log-Log plot of number of craters per bin, $N(D)\ vs\ D$(km), based on Barlow's Mars catalog. The number $N(D)$ is calculated by counting the number of craters in a bin $\Delta D = D_R - D_L$, and then dividing this number by the bin size. The point is placed at the mathematical average of $D$ in the bin: $(D_R + D_L)/2$. The bin size is $\Delta D = (\sqrt{2} - 1)D_L$, so that $\frac{D_R}{D_L} = \sqrt{2}$.

We can interpret the above formalism in a statistical or probabilistic manner. Thus, for instance, $\bar{\Phi}$ could be view as a probability of impacts per unit time, while $1/\tau_{mean}$ represents the probability,per unit time, for a crater to disappear. Accordingly, Eq.(1) is the familiar formula describing the evolution in time of $N(D)$, resulting from these production vs destruction processes. It is then important to evaluate the predicted probality of impacts, that can be confronted with observations. We do so next for Mars and also extend this application to Earth.

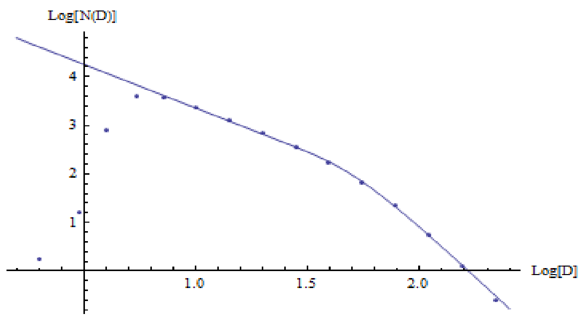


FIGURE (2): Comparing the model in Eqs. (1) to (4) with the Mars data in Figure (1).

We see from Eq. (4) that a numerical calculation of $\bar{\Phi}$ for Mars requires an estimate of $\tau_f$, so with that goal let us write $\tau_f$=(3.55$x10^3/\beta)my,\ my \equiv 10^6$ years, where $\beta$ is a number close to 1. For example, the range of values 3000my $< \tau_f <$ 4000my is covered by $\sim$0.9 $< \beta < \sim$1.2. Hence, from Eq. (4), we obtain: $\bar{\Phi} = \beta 10^6(D^{4.3}my)^{-1}$,and thus also find the cumulative rate: $\bar{\Phi}_C$(Mars)= $\int_D^\infty \bar{\Phi} dD = \beta x 10^6(3.3D^{3.3}my)^{-1}$. For instance, for $D = 20km$ we obtain: $\Phi_C$(Mars,$20km$)$\cong$ $15\beta(my)^{-1} \approx 15\ (my)^{-1}$, which implies that the cumulative flux per unit area is,:15 $/(4\pi R_m{}^2 my)$ $\cong$ $100x10^{-9}(mykm^2)^{-1}$, with $R_m$ being the Martian radius, The above results is considerably higher than the values for Earth given by Grieve and Shoemaker[4], namely:$(5.5 \mp 2.7)x10^{-9}(mykm^2)^{-1}$.Furthermore,for $D = 1km$, or, equivalently, impactor energies around a megaton, we have, $\bar{\Phi}_C$(Mars, $1km$)$\cong$ $(1/3.3years)$. It is not surprising that the impact rate on Mars is larger than that on Earth, because of Mars' proximity to the asteroid belt; however, the above conclusion shows that future Mars astronauts may have to deal with frequent damaging meteorite collisions. In particular, we expect that Mars visitors spending a few years there will have a high probability of witnessing a megaton-type meteorite impact. Moreover, these impacts are likely to cause more damage on the surface than on our planet, due to the much lower atmospheric Martian density.

Let us now study the implications for our planet of a flux of the form: $\Phi = A(D^{-4.3})$,corresponding to the cumulative flux: $\Phi_C(D) = \int_D^\infty \Phi dD = A/(3.3D^{3.3})$ . The value of $A$ can be estimated for Earth from the result of Grieve and Shoemaker[4] for $D = 20km$:
$\Phi_C(20km)$= $(5.5 \mp 2.7)10^{-9}(mykm^2)^{-1}\ 4\pi R^2 \approx$ $(2.8/my)[1 \mp 0.50\ ]$, where $R$ is the Earth's radius , and thus obtain: $A = 9.24(20)^{3.3}/my$, which implies

$$\Phi_C(D) = (2.8/my)[1 \mp 0.50]\ (20/D)^{3.3}]. \qquad 7$$

Table I illustrates the outcomes of the above formula for selected values of $D$. Note that $\Phi_C$ is the probable frequency of impacts, and $1/\Phi_C$ is the probable period between impacts, for diameters larger than or equal to $D$. It is interesting that $D = 5km$ has a statistical periodicity of about one in 3,680 years, suggesting that these are potentially historical events.

**Table I**

| D(KM) | $\Phi_{Acc}$(D)/(1$\mp$0.50) | $[\Phi_{Acc}(D)/(1 \mp 0.50)]^{-1}$ |
|---|---|---|
| 200 | 1/(712my) | 712my |
| 150 | 1/(275my) | 275my |
| 100 | 1/(72my) | 72my |
| 50 | 1/(7my) | 7my |
| 10 | 28/(my) | 35,700y |
| 5 | 272/(my) | 3,680y |

**Table II**

| $E$(megatons) | Approximate $d_m$ | Approximate $D$ | $[1 \mp 0.5]/\Phi_C$ |
|---|---|---|---|
| 250 | 160 | 4 | 1673 years |
| 100 | 120 | 3 | 761 " |
| 50 | 90 | 2.5 | 419 " |
| 20 | 70 | 2 | 191 " |
| 10 | 60 | 1.7 | 105 " |
| 5 | 40 | 1.4 | 58 " |
| 2 | 30 | 1.1 | 26 " |
| 1 | 26 | 0.93 | 14.5 " |
| 0.5 | 20 | 0.67 | 7.8 " |

The formation of craters with potential diameters of less than approximately $5km$ is strongly affected by the Earth's atmosphere, since these bodies can be fragmented or even disintegrated. Therefore,for $D < 5km$ we prefer to express the flux in terms of the kinetic energy, $E = (4\pi\rho/6)(d/2)^3v^2$ , and the diameter, $d$, of the impactor. To convert $D$ to $d$ we will use the Schmidt and Holsapple scaling equation: $D$=$10^{1.21a_1}d^{0.78a_2}$, where the values $a_1$ and $a_2$ are very close to 1, and we will put them as equal to 1. Table II gives values of $1/\Phi_C(E)$ for selected $E$, with $\rho = 2{,}400kg/m^3$ and $v = 20km/s$, and corresponding approximate values of $d$ in meters:$d_m$ and $D$.

**References:**[1] Barlow, N.G. (1988) Icarus, 75, 285.
[2]Bruckman W.,et al.,(2013),http://arxiv.org/ftp/arxiv/papers/1212/1212.3273.pdf.[3] Hartmann, W.K. ( 2002) Lunar and Planetary Science XXXIII, 1876.
[4]Grieve and Shoemaker, (1994):The Record of Past Impacts on Earth. In: "Hazards Due To Comets And Asteroids", T. Gehrels, Editor, The University Of Arizona Press.